ON THE ENHANCEMENT OF Λ_Q DECAY RATE

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The enhancement of the Λ_b and Λ_c decay rates due to four-quark operators is calculated. Hard gluon exchange modifies the wave function for the $b\bar{u}$ pair in Λ_b, $|\Psi(0)|^2_{bu}$, and the wave function for the $c\bar{d}$ pair in Λ_c, $|\Psi(0)|^2_{cd}$. The modified wave function is found to account for the discrepancy of 0.20 ps^{-1} among the lifetimes of Λ_b and B^0 and for the difference of 2.6 ps^{-1} in the case of Λ_c and D^0.

Heavy flavour hadrons, H, contain a heavy quark, Q (b and c), and a light cloud comprising of light quarks (anti-quarks) and gluons. When the mass of the heavy quark goes to infinity, the picture of the heavy hadron decays are so simplified that the light cloud has no role to play. Theoretical study of the heavy hadrons is facilitated by the expansion of the hadronic matrix element, based on the operator product expansion of QCD, in inverse powers of the heavy quark mass, m [1]. The leading order of the expansion, corresponding to the asymptotic limit of the heavy quark mass, describes the decay rate of a heavy flavour hadron as if it is of a free heavy quark decay. This implies the same lifetime for all heavy hadrons of a given heavy flavour quantum number. The next-to-leading order terms, appearing at $1/m^2$, describe the motion of the heavy quark inside the hadron and the chromomagnetic interaction, distinguish the lifetimes of the mesons on one hand and the baryons on the other (with exception of Ω_Q). The term corresponding to the third order in $1/m$ of the expansion is the matrix element of four-quark operators which contains effects such as W-exchange, weak annihilation and Pauli interference, coming from the spectator (light) quarks. At this order, the lifetimes of the various mesons differ among themselves and so is for baryons. The spectator effects which appear through four-quark operators, are supposed, yet poorly-understood, to explain the intricacies of the lifetime differences and to fix the lifetime hierarchy of the hadrons.

According to the idea of heavy quark expansion, all hadrons of a given heavy flavour are expected to have nearly the same lifetime. The theoretical prediction of the ratio of lifetimes of Λ_b and B^0, obtained to two orders in $1/m$, is 0.9. But this is found much higher than the observed value of the ratio: 0.78 for $\tau(\Lambda_b) = 1.20$ ps and for $\tau(B^0) = 1.58$ ps. On the other hand, the corresponding charm sector shows an intrinsically different picture due to the mass of the charm quark, which is not so asymptotically large as the b quark mass. In the charm case, the dominant effects come from the four-quark operators rather than the kinetic and chromomagnetic operators. Experimentally, the ratio of the lifetimes of $\tau(\Lambda_c)$ and $\tau(D^0)$ is 0.496 for $\tau(\Lambda_c) = 0.206$ ps and $\tau(D^0) = 0.415$ ps. Therefore, for both the cases, the explanation for the suspected small lifetime, and an equivalently enhanced decay rate, of Λ_Q should come from the third order term of the heavy quark expansion in $1/m$ where the matrix element involves four-quark operators. Given the present experimental decay rates for beauty hadrons: $\Gamma(\Lambda_b) = 0.83 \pm 0.02$ ps^{-1} and $\Gamma(B^0) = 0.63 \pm 0.05$ ps^{-1}, the needed enhancement is 0.2 ps^{-1}, whereas 2.6 ps^{-1} is needed for charmed hadrons of decay rates: $\Gamma(\Lambda_c) = 5$ ps^{-1} and $\Gamma(D^0) = 2.4$ ps^{-1}.

The four-quark operators are estimated using phenomenological models. Hence their size is questionable. Nevertheless, it is the only way now to do. The four-quark operators are related to the probability of finding the $Q\bar{q}$ pair at the origin simultaneously using quark models, denoted by $|\Psi(0)|^2_{Qq}$. The wave function is evaluated relating it to the mass splitting of hadrons arising out of the heavy-light quarks interaction. In Ref. [2], Rosner evaluated the wave function, using the hyperfine splitting in a similarly heavy-flavoured baryon, utilising the DELPHI value on Σ_b^* - Σ_b splitting [3] [1] Neubert and Sachrajda [4] introduced hadronic parameters accounting for hybrid renormalisation while parameterizing the four-quark matrix for b-flavoured hadrons. The hadronic parameters are yet unknown. Their values are obtained from QCD sum rules. As an extension to c-flavoured hadrons, Voloshin [5] studied the the charmed baryons. In this approach, the authors of Ref. [6] analysed the inclusive charmed-baryon decays to fix the hierarchy of charmed lifetimes.

In the present study, we take into account the contribution coming from the exchange of hard gluons when two different scales, the heavy quark mass, m, and the QCD scale, μ, are involved, while estimating the wave function in quark model. We treat the coupling constant corresponding to a meson and a baryon differently. It is found that the wave function, and hence the enhanced decay rate, is large in both the cases of b and c and explains part of the lifetime differences between Λ_b and B^0 as well as between Λ_c and D^0.

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[1]Though the DELPHI value has not hitherto been confirmed, its use does not alter the goal anyway since the central value of the mass splitting due to hyperfine interaction is expected to be around 50 MeV.

The enhancement in the decay rate of Λ_b is arising out of the processes involving four-quarks [4,7–10]: (a) the weak scattering process bu−− >cd in the Λ_b involving matrix elements between hadronic states of $(\bar{b}b)(\bar{u}u)$ operators and (b) the process contributing to the Pauli interference involving matrix elements of operators $(\bar{b}b)(\bar{d}d)$ operators. Hence, the enhancement of the Λ_b decay rate is given by:

$$\Delta\Gamma(\Lambda_b) = \frac{G_f^2}{(2\pi)}|\Psi(0)|_{bu}^2 |V_{ud}|^2 |V_{cb}|^2 m_b^2 (1-x)^2 [C_-^2 - (1+x)C_+(C_- - C_+/2)] \tag{1}$$

where $x = m_c^2/m_b^2$; C_- and $C_+ = C_-^{1/2}$ are the short distance QCD enhancement and suppression factors for quarks in a colour antitriplet and sextet respectively:

$$C_- = [\alpha_s(m_b^2)/\alpha_s(m_W^2)]^{4/\beta}, \quad \beta = 11 - 2n_f/3 \tag{2}$$

where n_f is the active quark flavours between m_b and m_W. The C_- term corresponds to the weak sattering process $bu \to cd \to bu$ and the other term represents the destructive interference between the two intermediate d quarks in the process $bd \to cudd \to cd$.

The wave function for the bu pair (in the initial baryon), $|\Psi(0)|_{bu}^2$, in Eq. (1) is of the form:

$$|\Psi(0)|_{bu}^2 = \frac{4}{3} \frac{\Delta M(B_{ijk})}{\Delta M(M_{i\bar{j}})} \xi |\Psi(0)|_{b\bar{u}}^2 \tag{3}$$

where

$$\Delta M(B_{ijk}) = \frac{16\pi}{9} \alpha_s \sum_{i>j} \frac{<S_i.S_j>}{m_i.m_j} |\Psi(0)|_{ij}^2 \tag{4}$$

and

$$\Delta M(M_{i\bar{j}}) = \frac{32\pi}{9} \alpha_s \frac{<S_i.S_j>}{m_i.m_j} |\Psi(0)|_{i\bar{j}}^2 \tag{5}$$

are the hyperfine mass splittings in a baryon and in a meson respectively. There is the colour factor 1/2 in the baryonic case due to the quark composition: a heavy quark and two light quarks. Under isospin symmetry, the effective masses of light quarks are equal. In the same token, wave functions for bu and bd pairs are equal. Equation (3) is obtained for the values of $<S_i.S_j> = (1/4, -3/4)$ with spin (0, 1) for the meson and $<S_i.S_j> = (1/4, -1/2)$ with spin (1/2, 3/2) for the baryon with $S_{qq} = 1$. In Eq. (3), ξ is the ratio of the coupling constants governing a baryon and a meson. The coupling governing a baryon is stronger than that of a meson. Though ξ has been chosen as a free parameter varying from 0.25 to 1.5, it can be exactly calculated [11]. In rigorous sense, it should be more than unity. The wave function on the right hand side of Eq. (3) corresponds to the matrix element for B-meson decay into vacuum, parameterised as,

$$| < 0|\bar{q}\gamma_\mu\gamma_5 Q|B > |^2 = f_B^2 M_B^2 \tag{6}$$

and to the relation obtained in the non-relativistic limit [12]

$$| < 0|\bar{q}\gamma_\mu\gamma_5 Q|B > |^2 = 12 M_B |\Psi(0)|_{b\bar{u}}^2 \tag{7}$$

Both Eqs. (6) and (7) characterise the same process involving a heavy quark and a light quark (and gluons) but are normalised at two different scales: the normalisation point of Eq. (6) is $\mu(\approx R^{-1})$ which is the order of the virtuality of the quarks inside the meson, and the scale of the Eq. (7) is the mass of the heavy quark. Therefore, in order to account for the hard gluon exchange between the light and heavy quarks, Eqs. (6) and (7) have to be related. The relation turns out to be an evolution equation of current operators [13,14]:

$$< 0|\bar{q}\gamma_\mu\gamma_5 Q|B > (m_Q) = < 0|\bar{q}\gamma_\mu\gamma_5 Q|B > (\mu)[\alpha_s(\mu)/\alpha_s(m_Q)]^{\gamma/\beta}, \quad \mu \ll m_Q \tag{8}$$

where $\gamma (= 2)$ is the hybrid anomalous dimension. The coupling constant defined through leading order renormalisation group equation can be expressed as

$$\alpha_s(m) = \frac{\alpha_s(\mu)}{[1 - \frac{b}{2\pi}\alpha_s(\mu)ln(\mu/m)]} \tag{9}$$

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Using eq. (8), one can get the eq. (3) of the form

$$|\Psi(0)|^2_{bu} = \frac{4}{3} \frac{\Delta M(B_{ijk})}{\Delta M(M_{i\bar{j}})} \frac{f_B^2 M_B}{12} \xi \left(1 - \frac{b}{12} \alpha_s(\mu) ln \frac{\mu}{m}\right)^{\frac{2\gamma}{\beta}}$$ (10)

The parameter ξ describes the ratio of the coupling constants of baryon and meson. The other term in the bracket represents the logrithmic effects due to the hard gluon exchange that would be expected when one goes down to a scale as small as the hadronic scale from the heavy quark mass. This obviously modifies the expectation values of the wave function density at the origin.

The choice of values of the parameters are given in Table I. The $\Delta M(B_{ijk})$ for Λ_b is given by M(Σ_b^*)- M(Σ_b) of DELPHI [3]. For c, the $\Delta M(B_{ijk})$ is given by M(Σ_c^*) = 2517.5 1.4 MeV [15,16] and M(Σ_c) = 2452.2 0.6 MeV [15].

The enhanced decay rate is now given by Eq. (3) alongwith eq. (10). The results for the enhanced decay rate of Λ_b and Λ_c are given in Tables II and III respectively. The wave function density is, roughly speaking, independent of the hadronic scale value. However, its dependence upon the parameter ξ is important. For ξ is graeter than one, the enhanced decay rate becomes closer to the enhancement required to explain the smaller lifetimes of Λ_b and Λ_c baryons.

In conclusion, it is demonstrated that the four-quark operators appearing at the third order in 1/m expansion explains the needed enhancement in the decay rates of Λ_Q. Though subtle, the couplings of meson and Λ_b baryon make much difference. On the other hand, if the couplings are considered equal, the four-quark operators still account for the difference in lifetimes the Λ_b baryon and B meson.

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TABLE I. Choice of values of parameters [15] ($n_f = 4$; $\Lambda_{QCD} = 0.2$ GeV).

Q	Δ M(B_{ijk})	Δ M($M_{i\bar{j}}$)	M$_H$	f$_H$	m$_Q$	α_s(m$_Q$)	C_+	C_-
b	0.056	0.046	5.2789	0.190	5.1	0.20	0.87	1.31
c	0.0653	0.142	1.8693	0.240	4.7	0.29	0.79	1.57

TABLE II. Enhanced Decay Rate for Λ_b. Needed Enhancement is 0.2 ps^{-1}.

| ξ | $\mu = 1$ GeV | | $\mu = 0.5$ GeV | |
	$|\Psi(0)|_{b\bar{d}}$ (10^{-2}GeV^3)	$\Delta\Gamma(\Lambda_b)$(ps^{-1})	$|\Psi(0)|_{b\bar{d}}$ (10^{-2}GeV^3)	$\Delta\Gamma(\Lambda_b)$(ps^{-1})
0.50	1.3154	0.0763	1.3422	0.0779
0.75	1.9732	0.1145	2.0133	0.1168
1.00	2.6309	0.1527	2.6844	0.1558
1.25	3.2886	0.1909	3.3555	0.1948
1.50	3.9463	0.2291	4.0266	0.2337

TABLE III. Enhanced Decay Rate for Λ_c. Needed Enhancement is 2.6 ps^{-1}.

| ξ | $\mu = 1$ GeV | | $\mu = 0.5$ GeV | |
	$|\Psi(0)|_{c\bar{d}}$ (10^{-3}GeV^3)	$\Delta\Gamma(\Lambda_c)$(ps^{-1})	$|\Psi(0)|_{c\bar{d}}$ (10^{-3}GeV^3)	$\Delta\Gamma(\Lambda_c)$(ps^{-1})
0.50	2.7718	1.2005	2.8216	1.2221
0.75	4.1578	1.8008	4.2325	1.8332
1.00	5.5437	2.4011	5.6433	2.4443
1.25	6.9297	3.0015	7.0542	3.0553
1.50	8.3156	3.6017	8.4650	3.6665